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Rayleigh streaming at large Reynolds number and its effect on shear flow

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A fluid contained between two parallel walls, one of which is at rest and the other moving in the longitudinal direction with a constant velocity, is examined when a standing sound wave is imposed in the transverse direction. Vortical acoustic streaming appears in the region between the walls. The streaming is not affected by the main flow. A qualitative analysis is presented for the Navier–Stokes equations governing the steady-streaming component of the motion. The study considers the case of flow with high streaming Reynolds number and makes an explicit determination of the vorticity in the inviscid core region. The effect of the streaming upon the shear flow in the longitudinal direction is then analysed asymptotically. A periodic structure of the wall shear stress in the transverse direction is detected in which vast areas of vanishing wall shear stress alternate with narrow regions where it increases significantly. A relation expressing the mean wall shear stress in terms of the streaming Reynolds number is derived. Results obtained show that acoustic streaming results in a marked enhancement of the mean wall shear stress at the walls.

1. Introduction

The interaction between sound waves and viscosity leads to the formation of secondary vortex flow. This acoustic streaming occurs when an acoustic standing wave is present in fluid adjacent to a solid wall or it results from the vibrations of a solid body adjacent to fluid at rest.

Acoustic streaming in a uniform duct was first analysed by Lord Rayleigh (1883). This work has been continued by Westervelt (1953), Nyborg (1953), and Schlichting (1955). The treatment ignored the effect of the fluid inertia on the streaming motions. Lighthill (1978) has emphasized the fundamental principle that it is the attenuation of acoustic energy flux that makes momentum flux available to force the streaming motion.

The flow is characterized by a streaming Reynolds number R_s based upon the time-independent component of the fluid velocity and a linear dimension of the acoustic boundary layer. Stuart (1966), in the context of the flow induced by a vibrating cylinder, first recognized the importance of this parameter and predicted that a jet-like streaming flow, along the axis of oscillations, would originate at the cylinder. Davidson & Riley (1972) did a flow visualization of this jet from which it was possible to make quantitative measurements of the jet flow. Bertelsen (1974) has presented an experimental investigation of high-Reynolds-number steady streaming for an oscillating circular cylinder in a finite domain with a boundary which is cylindrical in shape. Duck & Smith (1979) have considered the flow of a fluid between two cylinders, when the inner cylinder performs small harmonic oscillations. The asymptotic solution consists of an inviscid core region, which has closed streamlines with constant vorticity

(Batchelor 1956) and a closed boundary layer. Kim & Troesch (1989) were able to solve, using a finite-difference method, the problem of the streaming flow between two concentric cylinders where the inner square cylinder performs harmonic oscillations and the outer circular cylinder is at rest. Tatsuno & Bearman (1990) have experimentally investigated flow structures over a wide range of parameters and identified the different flow regimes, whilst Stansby & Smith (1991) investigated numerically the viscous forces on a circular cylinder in orbital flow for small values of amplitude and low frequency. Streaming induced by a sphere, due to a pulsating source, was considered by Wang (1982) for $R_s \ll 1$, and by Amin & Riley (1990) for $R_s \gg 1$. Secondary streaming in a narrow cell caused by a vibrating wall, when the dimension of the container is smaller than the sonic wavelength, was studied by Vainshtein, Fichman & Pnueli (1994).

The problem of heat transfer associated with acoustic streaming induced by an oscillating circular cylinder has been studied by Richardson (1967) and Davidson (1973). Heat transfer due to acoustic streaming from a sphere has been treated by Gopinath & Mills (1993), and across the ends of a Kundt tube by Gopinath & Mills (1994). The problem of the influence of Rayleigh's acoustic streaming on heat transfer between two parallel plates which are kept at different temperatures has been analysed by Gutfinger, Vainshtein & Fichman (1994), and by Vainshtein, Fichman & Gutfinger (1995). The case $R_s \ll 1, R_s Pr \gg 1$ was considered and the enhancement of heat transfer by sound waves revealed.

Secomb (1978) has examined the flow in a channel with pulsating walls. The study covers all values of the streaming Reynolds number, and gives us an example inviscid rotational streaming in a confined flow. The internal acoustic flow in a waveguide with a slowly varying height was analysed by Thompson (1984). The oscillatory flow in a tube of slowly varying cross-section was considered by Hall (1974), who prescribes an oscillating pressure difference of constant amplitude. The oscillatory viscous flow in a tapered channel under conditions of fixed stroke volume was analysed by Grotberg (1984). The flow in a pipe of circular cross-section which is coiled in a circle has been studied by Lyne (1970) who analysed the secondary streaming generated by centrifugal effects in the limit $R_s \rightarrow \infty$. It is found that in corners adjacent to stagnation points the flow is irrotational in character, and the perturbation vorticity is convected around the corner on the streamlines of the motion. The analysis follows closely that of Harper (1963), dealing with the wake behind a bluff body in a uniform stream, and Moore (1963), concerning the boundary layer on a spherical gas bubble.

For many-body problems Ingham, Tang & Morton (1990) have examined both numerically and experimentally the steady two-dimensional flow through a cascade of normal flat plates, whilst Yan, Ingham & Morton (1993) considered a cascade of circular cylinders which oscillate harmonically in an unbounded viscous fluid.

Traditional studies of acoustic shear flow interaction by Pridmore-Brown (1958), Mungur & Gladwell (1969), and Hersh & Catton (1971) examined the quasi-steady properties of sound waves propagating in fully developed shear flows above a flat surface or in a planar duct in the longitudinal direction. Solutions predict significant distortion of propagating wave modes as a result of acoustic refraction. Baum & Levine (1987) used numerical methods to solve an initial boundary-value problem based on the Reynolds-averaged Navier–Stokes equations in order to reveal mechanisms for energy exchange between the acoustic and mean flow fields. Acoustic shear flow interactions in a rectangular duct were studied by Wang & Kassoy (1992), using the idealized model as a paradigm to demonstrate the complex response of an initially steady flow to an imposed longitudinal velocity disturbance.

It is worth emphasizing that the works cited in the previous paragraph deal with the streamwise effect of a sonic wave on a main shear flow. We should note the paper by Jung, Mangiavacchi & Akhavan (1992) in which the response of wall-flow turbulence to high-frequency spanwise oscillations was investigated by direct numerical simulations of a planar channel flow subjected either to an oscillatory spanwise cross-flow or to the spanwise oscillatory motion of a channel wall. The results show that under certain conditions the turbulent bursting process was suppressed, leading to sustained reductions in the turbulent drag.

The present work considers simple shear flow between two parallel walls affected by a plane standing wave, imposed in the spanwise transverse direction. The interaction between the sound wave and the walls results in the appearance of secondary streaming periodic in that direction. Our main concern is with the case $R_s \gg 1$, where

$$R_s = \frac{3w_0^2 h^2 \omega}{32 \nu c^2}. \tag{1.1}$$

In (1.1) w_0 is a characteristic amplitude of oscillations, ω is a typical frequency, c is the speed of sound, ν is the kinematic viscosity, and h is the distance between the walls. The study is aimed at analysing the effect of this Rayleigh-type streaming on the wall shear stress distribution. However, the most important result is the determination of the secondary streaming itself at large values of R_s.

2. Formulation of the problem

One example of a viscous flow is discussed in this paper: the case in which an external plane standing sound wave is imposed in the transverse direction with respect to the main flow. Let the fluid be enclosed between two parallel walls, one of which, the (x', z')-plane, is at rest, while the other is moving in its plane in the longitudinal x'-direction with a constant velocity, U (figure 1). Such a formulation of the problem leads to a solution describing simple Couette flow. We assume that the velocity of the moving wall, U, is much smaller than that of sound,

$$U/c \ll 1. \tag{2.1}$$

This condition enables us to neglect the compressibility of the main flow. Let a plane standing wave be imposed in the z'-direction:

$$\left.\begin{aligned} W = w_0 \cos nz' \cos \omega t', &\quad -\infty < x' < \infty, \\ n = \omega/c, &\quad 0 < y' < h, \end{aligned}\right\} \tag{2.2}$$

where $W(z', t')$ is the external-flow velocity, t' is time, and y' is the normal coordinate. In the absence of a main flow in the longitudinal direction, these conditions lead to a solution corresponding to acoustic streaming which for small R_s was analysed by Rayleigh (1883). Hence, the flow under consideration is a superposition of simple shear flow on Rayleigh-type acoustic streaming. Let x', y', z' denote the coordinates, and u', v', w' the corresponding velocity components. Under the above conditions, none of the flow parameters depend on the x'-variable.

The properties of acoustic streaming are more typically seen when the characteristic length of the problem, i.e. the distance between the walls, is much smaller than the sonic wavelength, λ, but much larger than the thickness of the Stokes layer, $l = (2\nu/\omega)^{1/2}$:

$$\lambda \gg h \gg l. \tag{2.3}$$

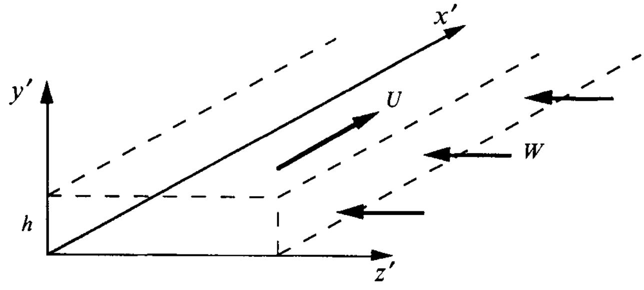

FIGURE 1. The flow domain with coordinates. Here h is the distance between the walls, U is the velocity of the upper wall, W is the velocity of the imposed standing sound wave.

In view of the second condition, we can distinguish in the flow region a narrow acoustic boundary layer adjacent to the walls in which the velocity decreases from its value in the sound wave to zero at the solid surface. Compressibility is ignored as the velocity in this layer is much smaller than that of sound, and the characteristic dimension, h, is much smaller than the wavelength:

$$w_0/c \ll 1, \quad \omega h/c \ll 1. \tag{2.4}$$

The equation of the acoustic boundary layer in the (y', z')-plane may be solved by successive approximations with respect to the small quantity w_0, the amplitude of the velocity fluctuations

$$w_0/\omega h \ll 1. \tag{2.5}$$

In the second approximation the right-hand side of the acoustic boundary-layer equation contains steady terms which give rise to the time-independent term in the velocity (Schlichting 1955)

$$w'_s = \frac{3w_0^2}{8c} \sin 2nz'. \tag{2.6}$$

The velocity does not vanish at large distances from the wall, i.e. outside the acoustic boundary layer. Equation (2.6) serves as a boundary condition on the Navier–Stokes equations.

For the longitudinal motion problem the leading-order velocity contribution is time independent and the transport of momentum across the narrow acoustic boundary layer is essentially due to viscous diffusion. The resistance to momentum transfer is negligible in this narrow region and the velocity of the main flow essentially does not change across the inner acoustic layer. This does not provide any information on the driving velocity gradient and attention is turned to the momentum transport effects in the outer region responsible for momentum transfer.

The equations of motion, determining the flow in question are as follows:

$$v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'} = \nu\left(\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right), \tag{2.7}$$

$$v'\frac{\partial v'}{\partial y'} + w'\frac{\partial v'}{\partial z'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'} + \nu\left(\frac{\partial^2 v'}{\partial y'^2} + \frac{\partial^2 v'}{\partial z'^2}\right), \tag{2.8}$$

$$v'\frac{\partial w'}{\partial y'} + w'\frac{\partial w'}{\partial z'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial z'} + \nu\left(\frac{\partial^2 w'}{\partial y'^2} + \frac{\partial^2 w'}{\partial z'^2}\right), \tag{2.9}$$

$$\frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0, \tag{2.10}$$

where ρ', p' are the fluid density and pressure. The relevant boundary conditions are

$$\left.\begin{array}{l} u' = v' = 0, \quad w' = w'_s \quad \text{at} \quad y' = 0, \\ u' = U, \quad v' = 0, \quad w' = w'_s \quad \text{at} \quad y' = h. \end{array}\right\} \tag{2.11}$$

The system of equations (2.7)–(2.10) may be decomposed in two parts. The first one includes (2.8)–(2.10) which do not contain the longitudinal velocity component, u'. Its solution describes secondary streaming which is not affected by the flow in the x'-direction. With the solution for velocity components v', w', equation (2.7) for u' may be analysed separately. This study enables us to reveal the influence of acoustic streaming upon simple Couette flow. Without solving the whole problem, we shall deal with the asymptotic analysis and evaluate the effect of the secondary vortical flow upon the shear stress σ'_{xy}, at the walls, $y' = 0$ and $y' = h$:

$$\sigma'_{xy} = \rho' \nu \left(\frac{\partial u'}{\partial y'}\right)_{y'=0}. \tag{2.12}$$

The mean value of the wall shear stress $\overline{\sigma'_{xy}}$ will be the main object of the investigation; here an overbar denotes an average with respect to the transverse variable, z'.

3. Statement of the problem of secondary streaming

As outlined in §2, we consider first the acoustic streaming at the (y', z')-plane. In terms of the stream function, ψ', where

$$w' = \partial\psi'/\partial y', \quad v' = -\partial\psi'/\partial z', \tag{3.1}$$

equations (2.8)–(2.10) and relevant boundary conditions may be rewritten in the form

$$\left.\begin{array}{l} -\dfrac{\partial\psi'}{\partial z'}\dfrac{\partial}{\partial y'}\nabla^2\psi' + \dfrac{\partial\psi'}{\partial y'}\dfrac{\partial}{\partial z'}\nabla^2\psi' = \nu\nabla^4\psi', \\ \dfrac{\partial\psi'}{\partial y'} = \dfrac{3w_0^2}{8c}\sin 2nz', \quad \dfrac{\partial\psi'}{\partial z'} = 0 \quad \text{at} \quad y' = 0, \quad y' = h. \end{array}\right\} \tag{3.2}$$

Let us introduce the vorticity Ω':

$$\Omega' = -\nabla^2\psi', \tag{3.3}$$

and the following non-dimensional variables:

$$y = \frac{2y'}{h} - 1, \quad z = 2nz', \quad \psi = \frac{32c}{3w_0^2 h}\psi', \quad \Omega = \frac{8ch}{3w_0^2}\Omega'. \tag{3.4}$$

The first condition in (2.3) may be rewritten in the form

$$\epsilon = nh \ll 1. \tag{3.5}$$

According to this condition, the derivatives with respect to y are much larger than those with respect to z. We may represent Ω in the form

$$\Omega = -\frac{\partial^2\psi}{\partial y^2} - \epsilon^2\frac{\partial^2\psi}{\partial z^2}. \tag{3.6}$$

Thus, (3.2) may be rewritten in the form

$$\left.\begin{array}{l} -\dfrac{\partial\psi}{\partial z}\dfrac{\partial\Omega}{\partial y} + \dfrac{\partial\psi}{\partial y}\dfrac{\partial\Omega}{\partial z} = R_s^{-1}\left(\dfrac{\partial^2\Omega}{\partial y^2} + \epsilon^2\dfrac{\partial^2\Omega}{\partial z^2}\right), \\ \dfrac{\partial\psi}{\partial y} = 2\sin z, \quad \dfrac{\partial\psi}{\partial z} = 0 \quad \text{at} \quad y = \pm 1, \end{array}\right\} \tag{3.7}$$

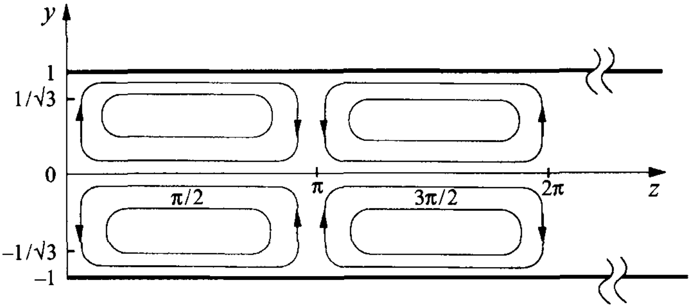

FIGURE 2. Rayleigh's acoustic streaming between the walls in terms of dimensionless variables. The field of view corresponds to one vortical cell.

where R_s is defined in (1.1). The problem first attracted the attention of Rayleigh (1883) who was concerned with creeping motions, $R_s \ll 1$. In this case the inertial terms in (3.7) are negligible compared with the principal viscous term. The simplest form of the Navier–Stokes equation

$$\frac{\partial^4 \psi}{\partial y^4} = 0, \quad \frac{\partial \psi}{\partial y} = 2 \sin z, \qquad \frac{\partial \psi}{\partial z} = 0 \quad \text{at} \quad y = \pm 1 \tag{3.8}$$

yields Rayleigh's solution

$$\psi = (y^3 - y) \sin z, \quad v = -\epsilon (y^3 - y) \cos z, \quad w = (3y^2 - 1) \sin z. \tag{3.9}$$

The flow described by this expression consists of two series of vortices lying symmetrically about the median plane, $y = 0$, and periodic in the z-direction, with period 2π as shown in figure 2. The normal velocity component is directed towards the walls at the antinode, $z = 0$, and away from them at the node, $z = \pi$. The contributions at $z = 2\pi$ are identical to those at $z = 0$. The periodicity of the solution results in the conditions

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{at} \quad z = 0, z = \pi. \tag{3.10}$$

The centres of the vortices are situated at the points $(\pm 1/\sqrt{3}, \pi/2), (\pm 1/\sqrt{3}, 3/2\pi)$ in the (y, z)-plane.

Our concern is with the case $R_s \gg 1$. The condition (3.10) holds in this case also, since the substitution $z = -z$ (or $z - \pi = \pi - z$), $\psi = -\psi$ does not change the equation and the boundary conditions in (3.7). From the obvious symmetry of the problem, the flow is symmetrical about the plane $y = 0$. Hence,

$$\frac{\partial \psi(y,z)}{\partial y} = \frac{\partial \psi(-y,z)}{\partial y}, \quad \frac{\partial \psi(y,z)}{\partial z} = -\frac{\partial \psi(-y,z)}{\partial z}. \tag{3.11}$$

From (3.10), (3.11) it follows that both components of the velocity vanish at the points $(\pm 1, 0), (\pm 1, \pi), (0, 0), (0, \pi)$. These points represent stagnation points of the flow field.

4. Analysis of acoustic streaming for $R_s \to \infty$

We now seek an asymptotic solution to (3.6), (3.7) in the limit $R_s \to \infty, \epsilon \to 0$. For such values of the flow parameters, the coefficients of highest-order derivatives in (3.7) are very small, therefore viscosity manifests itself only in narrow regions. Thin

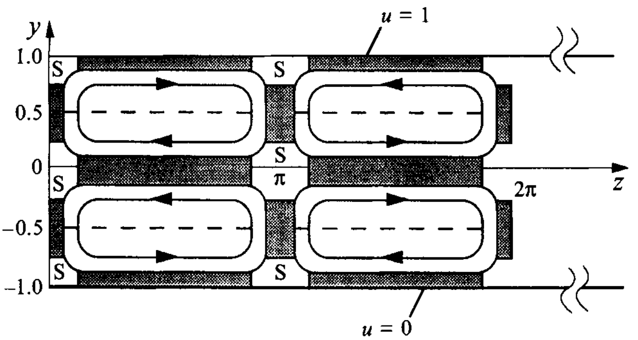

FIGURE 3. Acoustic streaming between the walls when R_s is large. Shaded regions denote boundary layers; unshaded regions have uniform vorticity; the corner regions near the stagnation points are denoted by the letter S.

boundary layers of thickness $O(R_s^{-1/2})$ will be formed at the walls within which, assuming that the layers do not separate, the velocity of the flow is adjusted to that dictated by the flow in the core of the domain. The anticipated solution of the problem as R_s increases behaves with R_s according to the order of magnitude associated with classical boundary-layer theory, while the core vorticity tends to a constant value, Ω_c say (Batchelor 1956), with the streamlines in the core flow becoming closed. A picture of the flow with a sketch of the streamlines is given in figure 3. The unshaded regions correpond to the core in which the flow has uniform vorticity. The vorticity in one core region has the same magnitude as, but the opposite sign to, the vorticity in the other core region. The regions in the vicinity of the stagnation points are denoted S. Boundary layers are also formed along the lines of symmetry: at line $y = 0$ of thickness $O(R_s^{-1/2})$ and at lines $z = 0$, π of thickness $O(\epsilon R_s^{-1/2})$. The fluid in the boundary layer at the wall in one core region meets boundary-layer fluid from the other core region, relating to the other wave or the other series of vortices. The two boundary layers impact, and must continue along the lines of symmetry. The velocity is continuous across the lines of symmetry but the vorticity is not, and the corresponding boundary layers represent the regions in which the discontinuity is smoothed out (Harper 1963).

The value of Ω_c is determined by the intensity of the imposed standing wave, and it has to be fixed by proper matching of the solution in the boundary layers and the core region.

4.1. *Motion in the core region*

We solve for the flow in the core region of the lower left rectangle in figure 3, i.e. for $-1 \leqslant y \leqslant 0, 0 \leqslant z \leqslant \pi$. For the other rectangles, the solution may be obtained using symmetry conditions (3.10), (3.11). If we refer to flow in the core by a subscript c, the governing equation for ψ_c in the core is

$$\frac{\partial^2 \psi_c}{\partial y^2} + \epsilon^2 \frac{\partial^2 \psi_c}{\partial z^2} = -\Omega_c, \tag{4.1}$$

where Ω_c is positive according to the velocity distribution on the lower wall. The boundary condition on ψ_c is

$$\psi_c = 0 \quad \text{at} \quad y = -1, 0 \quad \text{or} \quad z = 0, \pi. \tag{4.2}$$

The solution of (4.1) subject to (4.2) may be written down in the form of a double Fourier series. Fortunately, the presence of the small parameter ϵ in (4.1) allows one to obtain a simplified asymptotic solution justified in the regions close to antinode, $z=0$, or node, $z=\pi$, lines. For line $z=0$, the boundary condition is taken as follows:

$$\left.\begin{aligned} \psi_{ca} &= 0 \quad \text{at} \quad y=-1,0 \quad \text{or} \quad z=0, \\ \psi_{ca} &= O(1) \quad \text{as} \quad z\to\infty. \end{aligned}\right\} \tag{4.3}$$

We refer to flow in this region by a subscript *ca*. The solution of (4.1) subject to (4.3) is found to be

$$\psi_{ca} = \frac{4\Omega_c}{\pi^3}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^3}\left(\exp\left[-\frac{\pi(2m+1)}{\epsilon}z\right]-1\right)\sin(2m+1)\pi y. \tag{4.4}$$

For the region close to line $z=\pi$, the solution may be obtained from (4.4) by replacing z by $\pi-z$. In the limit $\epsilon\to 0$, (4.4) yields

$$\psi_c = -\tfrac{1}{2}\Omega_c y(1+y), \quad v_c = 0, \quad w_c = -\tfrac{1}{2}\Omega_c(1+2y). \tag{4.5}$$

These expressions describe the core solution in the region far from node or antinode lines. We can see that the transverse velocity component, w_c, changes sign at line $y=-0.5$ which is shown in figure 3 by a dashed line.

The solution (4.4) gives as the normal velocity component at the edge of the core

$$v_{ca}|_{z=0} = V_{ca} = \frac{4\Omega_c}{\pi^2}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}\sin(2m+1)\pi y. \tag{4.6}$$

For line $z=\pi$, the velocity component has the same magnitude and the opposite sign. In accordance with (4.6), the normal velocity distribution is symmetrical about line $y=-0.5$, and it has a minimum value at this line.

For the transverse velocity component at the edge of the core, the solution (4.4) yields

$$w_{ca}|_{y=-1,0} = W_{ca} = \mp\frac{4\Omega_c}{\pi^2}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}\left(\exp\left[-\frac{(2m+1)\pi}{\epsilon}z\right]-1\right), \tag{4.7}$$

where the signs minus and plus stand for $y=-1$ and $y=0$, respectively. In the limit $\epsilon\to 0$, we obtain

$$W_c = \pm\tfrac{1}{2}\Omega_c, \tag{4.8}$$

where now the signs plus and minus stand for $y=-1$ and $y=0$, respectively.

4.2. *The flow near the stagnation points*

According to (4.4), (4.6) the stream function ψ_{ca} in the vicinity of lines $z=0$ and $z=\pi$ may be represented in the form

$$\psi_{ca} = -\frac{z}{\epsilon}V_{ca} + O(z^2), \quad \psi_{ca} = -\frac{\pi-z}{\epsilon}V_{ca} + O((\pi-z)^2) \tag{4.9}$$

respectively.

The streamlines corresponding to the edge of the core diverge from the right-angled corners near the stagnation points. Therefore, (4.6), (4.7), (4.9) are not valid near these points (in fact, the coefficient of $\Omega_c z(y+1)$ in the expansion of (4.4) for small z and $(y+1)$ diverges). Thus the vorticity might be expected to fall to zero as the stagnation points are approached. Hence, the solution for the stream function of the irrotational flow at points sufficiently near stagnation point $(-1,0)$ can be found from (4.1), with the vorticity put equal to zero and boundary conditions (3.7), (3.10):

$$\psi = 2z(y+1), \quad v = -2\epsilon(y+1), \quad w = 2z. \tag{4.10}$$

This approximation holds up to the wall, since the solution also satisfies the viscous equation (3.7) with the slip condition at $y = -1$. In order to match the rotational core solution (4.9) and solution (4.10), corresponding to the irrotational flow, we have to follow the whole procedure for matching the solutions for the core flow, the corner flow and the flow in the boundary layers. A discussion of some aspects of this procedure is given in Harper (1963) where the boundary layers behind a bluff body are considered. It is found that the vorticity is dominated by an irrotational term near the stagnation point.

In (4.10) the rate of strain near the stagnation point equals -2. This is determined by the flow in the core region. For the stagnation point $(0,0)$, the rate of strain must have the same magnitude but the opposite sign because of the symmetry of the core flow about line $y = -0.5$. Hence, the frictionless flow near the point is given by the stream function

$$\psi = -2zy, \quad v = 2\epsilon y, \quad w = -2z. \tag{4.11}$$

In accordance with the analysis above, the stream function in the vicinity of the antinode and the node can be represented in the limiting form commonly adopted for studying the velocity near the point of attachment in natural convection flows. In this limit as $z \to 0$ or $z \to \pi$ the stream function is

$$\psi = zf(y), \quad \psi = (\pi - z)f(y) \tag{4.12}$$

respectively. The function $f(y)$ near $y = -1$ and $y = 0$ is determined by (4.10), (4.11), respectively, and outside the regions adjacent to the stagnation points by (4.9). It is a symmetrical function about $y = -0.5$. In view of the second condition in (3.11), $f(y)$ is an odd function. We note that the integral

$$S(y) = \int_{-1}^{y} f(y)\,\mathrm{d}y, \tag{4.13}$$

which is an even function, has a minimum value equal to zero at $y = -1$ and a maximum value at $y = 0$ as y varies in segment $[-1, 0]$.

In the limit $\epsilon \to 0$, (4.9) gives the principal contribution to the integral (4.13) as compared with that of (4.10), (4.11). On evaluating integral (4.13) by substituting (4.9), we obtain

$$S_{max} = \frac{8\Omega_c}{\epsilon\pi^3} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^3}. \tag{4.14}$$

The second derivatives of $S(y)$ at $y = -1$ and $y = 0$ may be evaluated by means of (4.10), (4.11):

$$S''(-1) = 2, \quad S''(0) = -2. \tag{4.15}$$

We shall use these results in the next subsection.

4.3. *Motion in the viscous boundary layers*

The procedure of matching the solutions for the core flows, the corner flows and the flow in the boundary layers must fix the value of Ω_c. We can employ an approximate method of solution for the boundary layer that was developed by Harper (1963), Moore (1963) and Lyne (1970). The method is based upon a linearization of the boundary-layer equation. In order to match solutions corresponding to the different boundary layers, the method assumes that the vorticity and velocity profiles are convected around each corner on the streamlines of the motion. This assumption enables us to fix the value of Ω_c. However, for our problem Ω_c may be found by a method developed in Batchelor (1956) without any linearization procedure.

In accordance with the method, we use the von Mises form of the boundary-layer equation with $\mathrm{d}\xi'$ representing a displacement along the streamline $\psi' = \text{const}$ and q' being the dimensional velocity in this direction. Then we take the line integral around a closed contour coinciding with a streamline. In cases in which the velocity outside the boundary layer is uniform the operation results in the condition (Batchelor 1956)

$$\oint q'^2 \,\mathrm{d}\xi' = \text{const} \tag{4.16}$$

throughout the boundary layer. In our case, the velocity of the main stream is uniform when we consider the boundary layers at lines $y = -1$ and $y = 0$ and it is non-uniform at lines $z = 0$ and $z = \pi$. However, the contribution to the integral that is made along that part of the contour is $O(h/\lambda)$ and therefore negligible. Hence, we obtain that the condition (4.16) holds in the present case.

On evaluating integral (4.16) for the streamline just outside the boundary layer, we find the constant in the right-hand side of (4.16) to be equal to $\pi\Omega_c^2/2$. On evaluating the integral for the streamline going along the wall, where $q = w_s = 2\sin z$, and the axis of symmetry $y = 0$, where $q = W_c = -\Omega_c/2$, we obtain $\Omega_c = 2\sqrt{2}$. The result is correct to leading order, given $h/\lambda \ll 1$.

We now study details of the viscous boundary layer at the wall and the axis of symmetry. Because the velocity varies very slightly across the layer, we may replace the nonlinear equation by a tractable linear equation (Moore 1963; Harper 1963; Lyne 1970).

The boundary-layer equation for the layer adjacent to the axis of symmetry $z = 0$ is

$$\frac{v}{\epsilon}\frac{\partial \Omega}{\partial y} + w\frac{\partial \Omega}{\partial z} = R_s^{-1}\epsilon^2\frac{\partial^2 \Omega}{\partial z^2}. \tag{4.17}$$

We assume that the flow field in the boundary layer is a small perturbation of the velocity in the core and write

$$\Omega = \Omega_c + \Omega_p, \quad v = V_{ca} + v_p, \quad w = -\frac{z}{\epsilon}\frac{\mathrm{d}V_{ca}}{\mathrm{d}y} + w_p, \tag{4.18}$$

where a subscript p denotes a perturbation quantity. Substituting (4.18) into (4.17) and neglecting quadratic terms in the perturbation quantities, we have, as the boundary-layer equation

$$\frac{V_{ca}}{\epsilon}\frac{\partial \Omega_p}{\partial y} - \frac{z}{\epsilon}\frac{\mathrm{d}V_{ca}}{\mathrm{d}y}\frac{\partial \Omega_p}{\partial z} = R_s^{-1}\epsilon^2\frac{\partial^2 \Omega_p}{\partial z^2}. \tag{4.19}$$

We now transform (4.19) into the diffusion equation by the use of the following transformation:

$$\eta = zV_{ca}R_s^{1/2}\epsilon^{-2}, \quad \zeta = \epsilon^{-1}\int_{y_0}^{y} V_{ca}\,\mathrm{d}y. \tag{4.20}$$

Here y_0 is a fixed point, to be chosen so that ζ is positive everywhere along the layer under consideration. It may be taken quite close to zero but not zero since point $(0, 0)$ is a stagnation point. Equation (4.19) becomes

$$\frac{\partial \Omega_p}{\partial \zeta} = \frac{\partial^2 \Omega_p}{\partial \eta^2} \tag{4.21}$$

with the conditions
$$\left.\begin{aligned} &\Omega_p \to 0 \quad \text{as} \quad \eta \to +\infty, \\ &\Omega_p = -\Omega_c \quad \text{at} \quad \eta = 0, \\ &\Omega_p = \Omega_p(\eta, 0) \quad \text{at} \quad \zeta = 0. \end{aligned}\right\} \tag{4.22}$$

Here $\Omega_p(\eta, 0)$ is some initial condition, so that $\Omega_p(0,0) = -\Omega_c$. The solution of (4.21) subject to the above conditions is given in Carslaw & Jaeger (1959):

$$\Omega_p = \frac{1}{2(\pi\zeta)^{1/2}} \int_0^\infty \Omega_p(\eta', 0)\{\mathrm{e}^{-(\eta-\eta')^2/4\zeta} - \mathrm{e}^{-(\eta+\eta')^2/4\zeta}\}\,\mathrm{d}\eta' - \Omega_c \operatorname{erfc}\frac{\eta}{2\zeta^{1/2}}. \tag{4.23}$$

Since $\Omega_c = 2\sqrt{2}$, the perturbation cannot be small near $\eta = 0$, but we hope this will not alter the result qualitatively. Integrating once with respect to η one can obtain the solution for the perturbation of the normal velocity component.

The linearized boundary-layer equation for the layer along the line $y = 0$ can be written as

$$W_{ca}\frac{\partial \Omega_p}{\partial z} - \epsilon y \frac{\mathrm{d}W_{ca}}{\mathrm{d}z}\frac{\partial \Omega_p}{\partial y} = R_s^{-1}\frac{\partial^2 \Omega_p}{\partial y^2}, \tag{4.24}$$

with the conditions
$$\left.\begin{aligned} &\Omega_p \to 0 \quad \text{as} \quad y \to -\infty, \\ &\Omega_p = -\Omega_c \quad \text{at} \quad y = 0, \\ &\Omega_p = \Omega_p(y, z_0) \quad \text{at} \quad z = z_0. \end{aligned}\right\} \tag{4.25}$$

Here $\Omega_p(y, z_0)$ is some initial condition, so that $\Omega_p(0, z_0) = -\Omega_c$. Using the relevant transformation, which is similar to (4.20), we can transform equation (4.24) into the diffusion equation. This equation, under conditions (4.25), has a solution which is similar to (4.23).

The linearized boundary-layer equation for the layer adjacent to the wall, where the slip condition (3.7) is applied, is (Lyne 1970)

$$W_{ca}\frac{\partial w_p}{\partial z} - \epsilon(y+1)\frac{\mathrm{d}W_{ca}}{\mathrm{d}z}\frac{\partial w_p}{\partial y} + \epsilon w_p \frac{\mathrm{d}W_{ca}}{\mathrm{d}z} = R_s^{-1}\frac{\partial^2 w_p}{\partial y^2}, \tag{4.26}$$

with the conditions
$$\left.\begin{aligned} &w_p \to 0 \quad \text{as} \quad y \to \infty, \\ &w_p = 2\sin z - \tfrac{1}{2}\Omega_c \quad \text{at} \quad y = -1, \\ &w_p = w_p(y, z) \quad \text{at} \quad z = z_1. \end{aligned}\right\} \tag{4.27}$$

Here $w_p(y, z_1)$ is some initial condition, so that $w_p(-1, z_1) = 2\sin z_1 - \frac{1}{2}\Omega_c$. We can transform (4.26) into the diffusion equation by means of the transformation used in Lyne (1970). However, in the limit $\epsilon \to 0$, when $W_{ca} = W_c$ (see (4.8)), equation (4.26) may be simplified to

$$\frac{\Omega_c}{2}\frac{\partial w_p}{\partial z} = R_s^{-1}\frac{\partial^2 w_p}{\partial y^2}. \tag{4.28}$$

The solution of (4.28) subject to conditions (4.27) is given in Carslaw & Jaeger (1959):

$$\begin{aligned} w_p = {} & \frac{4}{\pi^{1/2}} \int_{\frac{y+1}{2[a(z-z_1)]^{1/2}}}^\infty \sin\left(z + \frac{(y+1)^2}{4a\mu^2}\right)\mathrm{e}^{-\mu^2}\,\mathrm{d}\mu - \tfrac{1}{2}\Omega_c \operatorname{erfc}\frac{y+1}{2[a(z-z_1)]^{1/2}} \\ & + \frac{1}{2[\pi a(z-z_1)]^{1/2}} \int_0^\infty w_p(y', z_1)\{\mathrm{e}^{-(y-y')^2/4a(z-z_1)} - \mathrm{e}^{-(y+y')^2/4a(z-z_1)}\}\,\mathrm{d}y'. \end{aligned} \tag{4.29}$$

Here we use the notation $a = 2/R_s \Omega_c$. The solution contains the transient terms and the Stokes periodic term which determines the behaviour of the solution as z increases.

5. Analysis of shear flow

Let us consider equation (2.7) for the velocity component u', in which the velocity components v', w' are assumed to be determined by the acoustic streaming analysed in §4. Going over to the dimensionless variables (3.4) and taking advantage of the condition (3.5), one obtains

$$\left.\begin{aligned} v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} = R_s^{-1}\frac{\partial^2 u}{\partial y^2}, \\ u=0 \quad \text{at} \quad y=-1, \quad u=1 \quad \text{at} \quad y=1, \end{aligned}\right\} \tag{5.1}$$

where $u = u'/U$ is the dimensionless velocity component. For large values of R_s, the coefficient of the highest-order derivative in (5.1) is a small parameter of the problem, therefore, viscosity has an effect only in boundary layers of thickness $O(R_s^{-1/2})$. The solution for u in the inviscid core represents the straight lines u = const; in the regions adjacent to the walls the expressions for u are $u = 0$ and $u = 1$. At $y = 0$ there is a discontinuity in the core solution. Viscosity manifests itself in several regions. A boundary layer is formed along plane $y = 0$ in which the discontinuity is smoothed out. The effect of viscosity is pronounced in the narrow regions near the stagnation points on the (y, z)-plane, i.e. in the vicinity of singular lines $(x, \pm 1, 0)$, $(x, \pm 1, \pi)$, $(x, 0, 0)$, $(x, 0, \pi)$ extended in the x-direction. It is worth recalling that the flows along these lines are essentially frictionless with respect to the transverse direction. The boundary layers are formed along planes $y = \pm 0.5$, corresponding to the dashed lines in figure 3, extended in the x-direction, since velocity components v, w vanish on these planes. It is clear that the contribution to the mean value of the wall shear stress in question can come from viscous forces only in the neighbourhood of those singular lines indicated above, which are adjacent to the walls, i.e. at the lines $(x, \pm 1, 0)$, $(x, \pm 1, \pi)$. Outside these lines, the wall shear stress tends to zero as R_s tends to infinity. The vanishing of the wall shear stress is related to the slip condition realized in acoustic streaming. Fluid particles move along the walls without losing any momentum. Recall that the resistance to momentum transfer is negligible in the inner acoustic layer. Hence, at large R_s the wall shear stress of the longitudinal motion approximately equals zero everywhere except in the vicinity of the singular lines. Figure 4(*a*) is a sketch of the velocity distribution outside such lines, for example, when $z = \frac{1}{2}\pi$. The regions of a steep change of the velocity in the figure correspond to planes $y = \pm 0.5$.

In order to estimate the mean value of the shear stress, $\overline{\sigma'_{xy}}$, let us treat equation (5.1) in the vicinity of the antinode, $z = 0$, and the node, $z = \pi$. In accordance with (3.1), (4.12), the velocity component w vanishes at these planes, and equations (5.1) become

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial y^2} = \pm R_s f(y)\frac{\partial u}{\partial y}, \\ u=0 \quad \text{at} \quad y=-1, \quad u=1 \quad \text{at} \quad y=1, \end{aligned}\right\} \tag{5.2}$$

where signs plus and minus denote the node and the antinode, respectively.

At $z = \pi$ the solution of the problem is given by

$$u = \frac{1}{F_1(R_s)}\int_{-1}^{y} e^{R_s S(y)}\,dy, \quad F_1(R_s) = \int_{-1}^{1} e^{R_s S(y)}\,dy. \tag{5.3}$$

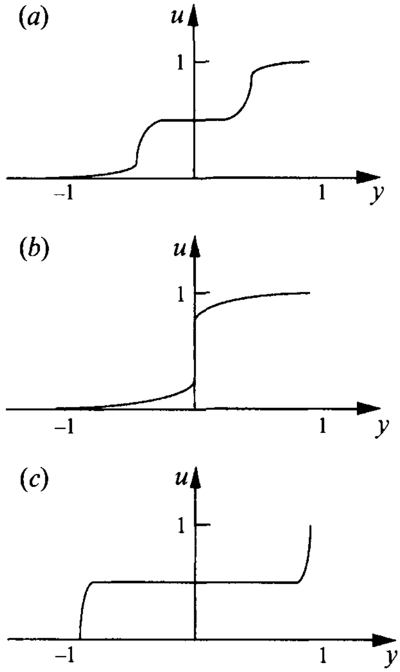

FIGURE 4. A sketch of the longitudinal velocity distribution at $R_s \gg 1$; (a), (b), (c) are in the neighbourhood of planes $z = \pi/2$, $z = \pi$ and $z = 0$, respectively.

Since $S(y)$ is an even function, the integral $F_1(R_s)$ can be represented in the form

$$F_1(R_s) = 2\int_{-1}^{0} e^{R_s S(y)}\,dy. \tag{5.4}$$

Since $S(y)$ has a maximum value at $y = 0$ given by (4.14), the integrand has a maximum value at this point. With (4.14), (4.15) the integral $F_1(R_s)$ at $R_s \gg 1$ may be estimated by the Laplace method (Olver 1974). The principal term of the asymptotic solution is

$$F_1(R_s) = (\pi/R_s)^{1/2}\, e^{R_s S_{max}}, \quad R_s \gg 1. \tag{5.5}$$

Let us introduce the velocity gradient g:

$$g(y,z) = \partial u/\partial y. \tag{5.6}$$

According to (5.3), (5.5) the velocity gradient near the walls is

$$g(\pm 1, \pi) = (R_s/\pi)^{1/2}\, e^{-R_s S_{max}}. \tag{5.7}$$

One can see that this term vanishes as R_s tends to infinity. At the same time the velocity gradient at $y = 0$ tends to infinity. Figure 4(b) is a sketch of the velocity distribution at the node, $z = \pi$.

At $z = 0$ the solution of the problem is given by

$$u = \frac{1}{F_2(R_s)}\int_{-1}^{y} e^{-R_s S(y)}\,dy, \quad F_2(R_s) = 2\int_{-1}^{0} e^{-R_s S(y)}\,dy. \tag{5.8}$$

Since $S(y)$ has a minimum value equal to zero at $y = -1$, the integrand has a maximum value at this point. With (4.15) the integral $F_2(R_s)$ at $R_s \gg 1$ may be estimated by the Laplace method. The principal term of the asymptotic solution is

$$F_2(R_s) = (\pi/R_s)^{1/2}, \quad R_s \gg 1. \tag{5.9}$$

The velocity gradient near the walls is

$$g(\pm 1, 0) = (R_s/\pi)^{1/2}. \tag{5.10}$$

One can see that this term tends to infinity as R_s tends to infinity. At the same time the velocity gradient at $y = 0$ tends to zero. Figure 4(*c*) is a sketch of the velocity distribution at the antinode, $z = 0$.

We now study the behaviour of the velocity gradient at points just outside singular line $z = 0$. Substituting (4.8) and $\Omega_c = 2\surd 2$ into (5.1), we arrive at the diffusion equation for u. The velocity gradient g also satisfies the diffusion equation

$$\frac{\partial g}{\partial z} = k\frac{\partial^2 g}{\partial y^2}, \tag{5.11}$$

where $k = R_s^{-1}/\surd 2$. We solve (5.11) subject to the initial condition $g(y, 0) = g_0(y)$, where $g_0(y)$ is the solution following from (5.8). Noting that function $g_0(y)$ may be defined to be equal to zero outside segment $[-1, 1]$, we obtain the solution

$$g(-1, z) = \frac{1}{2(\pi k z)^{1/2}}\int_{-\infty}^{\infty} g_0(y')\exp\left[-\frac{(y'+1)^2}{4kz}\right]\mathrm{d}y'. \tag{5.12}$$

Let us define an average velocity gradient at the walls:

$$\bar{g}_{y=-1} = \frac{1}{\pi}\int_0^{\pi} g(-1, z)\,\mathrm{d}z. \tag{5.13}$$

We can see that $k \rightarrow 0$ as $R_s \rightarrow \infty$, and therefore kz is close to zero as z varies in the whole segment $[0, \pi]$. Substituting (5.12) into (5.13) and replacing the integrand by a δ-function, we obtain approximate asymptotic relation $\bar{g}_{y=-1} = g_0(-1)$. Hence, at large R_s the mean wall shear stress is determined by the flow in the vicinity of the antinode.

We now define the dimensionless mean wall shear stress:

$$\lambda_w = \frac{2\overline{\sigma'_{xy}}}{\rho' U^2}. \tag{5.14}$$

Then the ratio of λ_w corresponding to (5.10) and that of the usual simple shear flow, denoted λ_{w0}, is given by

$$\frac{\lambda_w}{\lambda_{w0}} = \frac{2}{\pi^{1/2}}R_s^{1/2}, \quad R_s \gg 1. \tag{5.15}$$

Thus, for large R_s this ratio and, therefore, the increase of the wall shear stress due to acoustic streaming are proportional to the amplitude and to the square root of the frequency of acoustic oscillations.

As it has been pointed out above, the transport of momentum across the boundary layers adjacent to planes $y = 0$ and $y = \pm 0.5$ does not contribute directly to the mean value of the wall shear stress. Using (4.5), (5.1) one can arrive at the diffusion equation describing the flows in the boundary layers and write down the corresponding asymptotic solutions.

6. Conclusion

An external plane standing wave is imposed in the transverse direction with respect to a simple Couette flow. The statement of the problem assumes that none of the flow parameters depend on the longitudinal variable. The effect of the sonic wave results in the appearance of secondary streaming periodic in the transverse direction. The steady-state two-dimensional Navier–Stokes equation describing that acoustic streaming is analysed for large values of the streaming Reynolds number, R_s. The asymptotic solution consists of inviscid core regions (the majority of the flow field), which have closed streamlines and uniform vorticity; boundary-layer regions, not all of which are connected to the walls; and the corner regions near the stagnation points, in which the flows are frictionless in character. The approximate matching of the solutions in these regions fixes the value of the uniform vorticity.

The results obtained concerning the streaming flow field are then used to investigate the influence of the secondary streaming upon the shear flow. The steady-state equation for the longitudinal velocity component, including the inertial and viscous terms, is treated by the method of asymptotic expansions. Special attention is paid to the wall shear stress distribution. It is revealed that the distribution has periodic structure with respect to the transverse direction. The wall shear stress increases significantly in the neighbourhood of the singular lines relating to the antinodes of the imposed sound wave and extended in the longitudinal direction. Outside these lines, the wall shear stress vanishes. The lines correspond to the stagnation points of the transverse acoustic streaming; therefore, the flow near the lines is frictionless with respect to the transverse direction. An asymptotic relation for $R_s \gg 1$ expressing the averaged wall shear stress in the transverse direction is derived. It is shown that the increase of the mean wall shear stress due to acoustic streaming is proportional to the amplitude and to the square root of the frequency of the acoustic oscillations.

It seems that the periodic structure of the wall shear stress that has been detected may be of use in interpreting the bursting phenomenon in wall turbulent shear flow.

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